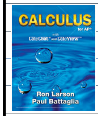

4.5 Integration By Substitution

(Day 1)

Objective: 1. Use Sustitution to Integrate.

Review:

1. $\frac{d}{dx}\left(\left(x^2+3x\right)^3\right) = 3\left(x^2+3x\right)^2(2x+3)$

2. $\frac{d}{dx}\left(\sin\left(e^x+4x\right)\right) = \cos\left(e^x+4x\right)\left(e^x+4\right)$

$$\boxed{\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)}$$

or

$$\frac{d}{dx}\left(f(u)\right) = f'(u)\,u'$$

CHAIN RULE

Thus. $\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$

$\int f'(u)\,u'\,dx = f(u) + C$

INTEGRATE.

1. $\int \left(x^2+1\right)^2 2x\,dx = \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \frac{1}{3}\left(x^2+1\right)^3 + C$

2. $\int \cos(4x)\,4\,dx = \int \cos u\,du$

$u = 4x$ $\quad = \sin u + C$

$du = 4\,dx$ $\quad = \sin(4x) + C$

3. $\frac{1}{3}\int \left(x^3+4\right)^5 3x^2\,dx = \frac{1}{3}\int u^5\,du$

$u = x^3+4$ $\quad = \frac{1}{3}\left(\frac{1}{6}\right)u^6 + C$

$du = 3x^2\,dx$ $\quad = \frac{1}{18}\left(x^3+4\right)^6 + C$

4. $\int \tan x \sec^2 x\,dx = \int u\,du$

$u = \tan x$ $\quad = \frac{1}{2}u^2 + C$

$du = \sec^2 x\,dx$ $\quad = \frac{1}{2}\left(\tan x\right)^2 + C$

5. $\int 5x^2\sqrt{x^3+1}\,dx$

$\left(\frac{1}{3}\right)5\int 3x^2\sqrt{x^3+1}\,dx = \frac{5}{3}\int u^{\frac{1}{2}}\,du$

$u = x^3+1$ $\quad = \frac{5}{3}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C$

$du = 3x^2\,dx$ $\quad = \frac{10}{9}\left(x^3+1\right)^{\frac{3}{2}} + C$

6. $\int \frac{\cos x}{\sin^2 x}\,dx$

$u = \cos x$ $\quad$ This will not work

$du = -\sin x\,dx$

$\int \frac{\cos x}{(\sin x)^2}\,dx = \int \frac{1}{u^2}\,du$

or

$u = \sin x$ $\quad = \int \frac{du}{u^2}$

$du = \cos x\,dx$

$= \int u^{-2}\,du$

$= -u^{-1} + C$

$= -\frac{1}{u} + C$

$= -\frac{1}{\sin x} + C$

Pg 341 # 1-25 odds