

CALCULUS
4.4
The Second Fundamental Theorem of Calculus

Objectives:

1. Understand and use the Second Fundamental Theorem of Calculus.
2. Use u-substitution in the Second Fundamental Theorem of Calculus.
3. Separate the integral and use the second Fundamental Theorem of Calculus.

Review:

1. $f(x) = \int_2^x t^3 dt$
 $f'(x) = x^3$

2. $f(x) = \int_3^x \sqrt{t} dt$
 $f'(x) = \sqrt{x}$

3. $f(u) = \int_2^u t^4 dt$
 $f'(u) = u^4$

4. $f(u) = \int_2^u \sin t dt$
 $f'(u) = \sin u$

5. $f = \int_2^u t \cdot t dt$
 $\frac{df}{du} = 2u$

6. $f = \int_2^u y e^y dy = -\int_{y=2}^u \frac{1}{y^2} e^y dy$
 $\frac{df}{du} = -\frac{1}{u^2} e^u$ $\frac{df}{du} = -\frac{1}{u^2} e^u$
 $\frac{df}{du} = -\frac{1}{u^2} e^u$ $\frac{df}{du} = -\frac{1}{u^2} e^u$

7. $f = \int_2^u \ln t dt$
 $\frac{df}{du} = \ln u$

$f'(x) = \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

8. $\frac{d}{dx} \left(\int_2^{x^2} t dt \right) = \frac{d}{du} \left(\int_2^u t dt \right) \cdot \frac{du}{dx}$
 $u = x^2$
 $\frac{du}{dx} = 2x$
 $= \frac{df}{du} \cdot \frac{du}{dx}$
 $= u \cdot \frac{du}{dx}$
 $= x^2 \cdot 2x$
 $= 2x^3$

Check: $\int_2^{x^2} t dt = \frac{1}{2} (t^2)_2^{x^2}$
 $= \frac{1}{2} (x^4 - 2^2)$
 $= \frac{1}{2} (x^4 - 4)$
 $f(x) = \frac{1}{2} x^4 - 2$
 $f'(x) = 2x^3$

9. $\frac{d}{dx} \int_0^{3x} \sqrt{1+t^3} dt = \frac{d}{du} \left(\int_0^u \sqrt{1+t^3} dt \right) \cdot \frac{du}{dx}$
 $u = 3x$
 $\frac{du}{dx} = 3$
 $= \sqrt{1+0^3} \cdot 3$
 $= \sqrt{1+27x^3}$

10. $f(x) = \int_2^x (4t+1) dt$

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$\int_2^{x+1} (4t+1) dt = \int_2^x (4t+1) dt + \int_x^{x+1} (4t+1) dt$
 $= \int_2^x (4t+1) dt + \int_x^{x+1} (4t+1) dt$
 $u = x+1$
 $\frac{du}{dx} = 1$
 $= (4(x+1)+1) \cdot \frac{du}{dx} - (4x+1)$
 $= (4x+5) \cdot 1 - (4x+1)$
 $= 4x+5 - 4x-1$
 $= 4$

11. $f(x) = \int_x^c \sqrt{t} dt + \int_c^{3x^2} \sqrt{t} dt$
 $f'(x) = \sqrt{3x^2} \cdot (2x) - \sqrt{x}$
 $= \sqrt{3x^2} \cdot 2x - \sqrt{x}$
 $= 12x^2 \sqrt{3x^2} - \sqrt{x}$

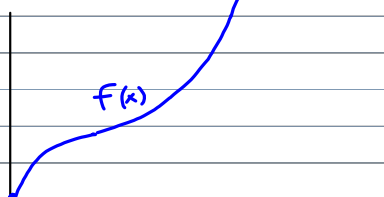
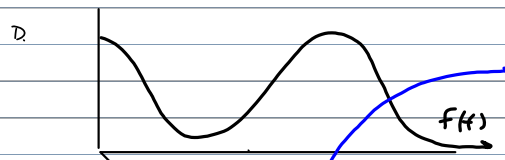
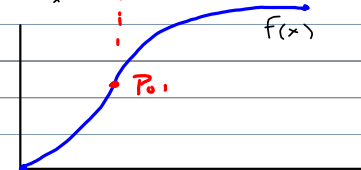
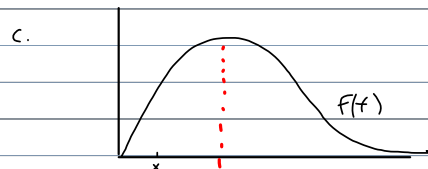
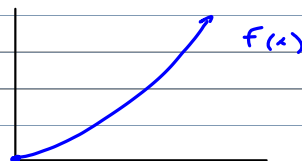
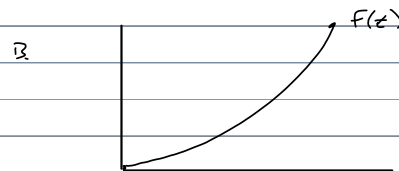
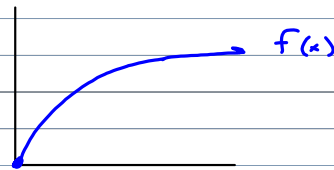
12. $f(x) = \int_2^x \ln t dt$ $f_{-2} f(x)$
 $f(x) = \int_2^x \ln t dt + \int_c^{2x^2} \ln t dt$

$$= \int_c^{5x^2} \ln t \, dt - \int_c^x \ln t \, dt$$

$$= \ln(5x^2)(5x^2) - \ln x$$

$$= 5x^2 \ln(5x^2) - \ln x$$

12 LET $f(x) = \int_0^x f(t) \, dt$
 GRAPH $f(x)$



HW Pg 330 # 85-97 odds

