

**3.6**  
(Day 1)

**Optimization Problems**

**Objective: 1. Solve Optimization Problems**

GLOBAL MAX/MIN Points occur AT CRITICAL NUMBERS AND END POINTS

CRITICAL NUMBERS occur where  $f'(x) = 0$  OR  $f'(a) = \text{undefined}$

WE ARE LOOKING FOR GLOBAL MAX/MIN POINTS!

1. Find the MAXIMUM AREA of a RECTANGLE when the PERIMETER is 100 ft.

PRIMARY EQUATION  
THE EQUATION YOU WANT TO MAXIMIZE OR MINIMIZE

SECONDARY EQUATION  
NEEDED IF YOU HAVE MORE THAN ONE INDEPENDENT VARIABLE

$A = xy$

$P = 2x + 2y$   
 $100 = 2x + 2y$   
 $50 = x + y$   
 $50 - x = y$  SUBSTITUTE

$A = x(50 - x)$  SIMPLIFY!!!

$A = 50x - x^2$  CRITICAL NUMBER IS 0

$A' = 50 - 2x$  MAX.

$0 = 50 - 2x$

$2x = 50$

$x = 25$        $y = 50 - x = 25$

ANSWER THE PROBLEM!

$A = xy$   
 $A = (25)(25)$   
 $A = 625 \text{ ft}^2$

2. Find two POSITIVE NUMBERS whose PRODUCT is 16 AND whose SUM is a MINIMUM.

PRIMARY EQUATION

$S = x + y$

$S = x + \frac{16}{x}$   
 $S' = 1 - \frac{16}{x^2}$   
 $0 = 1 - \frac{16}{x^2}$   
 $\frac{16}{x^2} = 1$   
 $x = 4$

SECONDARY EQUATION

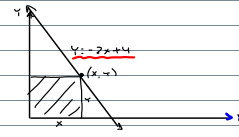
$P = xy$   
 $16 = xy$   
 $\frac{16}{x} = y$  SUBSTITUTE

$x = 4$

$x = 16$

$x = 4$      $y = \frac{16}{x}$   
 $y = 4$

$(4, 4)$

3. 

MAXIMIZE THE AREA OF THE RECTANGLE.

PRIMARY EQUATION

$A = xy$

SECONDARY EQUATION

$y = -2x + 4$

SUBSTITUTE

$A = x(-2x + 4)$  SIMPLIFY

$A = -2x^2 + 4x$  MAX.

$A' = -4x + 4$

$0 = -4x + 4$

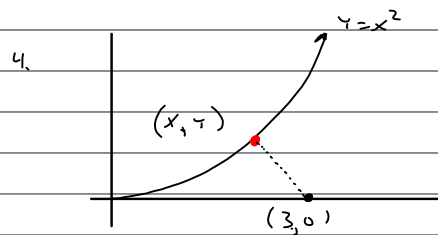
$4x = 4$

$x = 1$        $y = -2x + 4$   
 $y = 2$

AREA =  $xy$   
AREA = 2

## POSSIBLE STEPS TO SOLVE OPTIMIZATION PROBLEMS

1. IF POSSIBLE, DRAW A PICTURE AND ASSIGN VARIABLES
2. FIND THE PRIMARY EQUATION
3. IF NEEDED, FIND A SECONDARY EQUATION
4. SUBSTITUTE THE SECONDARY EQUATION INTO THE PRIMARY EQUATION SO THAT YOU ARE LEFT WITH ONE INDEPENDENT VARIABLE
5. IF NEEDED, SIMPLIFY !!
6. FIND THE CRITICAL NUMBERS
7. DETERMINE THE ENDPONTS
8. FIND THE GLOBAL MAX/MIN THAT IS NEEDED TO SOLVE THE PROBLEM
9. SOLVE THE PROBLEM !!



FIND THE POINT THAT WILL MINIMIZE  
THE DISTANCE FROM  $y = x^2$  TO THE  
POINT  $(3, 0)$

PRIMARY EQUATION                      SECONDARY EQUATION  
 $d = \sqrt{(x-3)^2 + (y-0)^2}$                        $y = x^2$

$$d = \sqrt{(x-3)^2 + (x^2)^2}$$

$$d = \sqrt{(x-3)^2 + x^4}$$

$$d' = \frac{1}{2}((x-3)^2 + x^4)^{-\frac{1}{2}} (2(x-3) + 4x^3)$$

$$\left( 0 = \frac{(x-3) + 2x^3}{\sqrt{(x-3)^2 + x^4}} \right) \sqrt{(x-3)^2 + x^4}$$

$$0 = 2x^3 + x - 3$$

$$1 = x$$

$$x = 1 \quad y = 1$$

Pg 202 # 5, 7, 9, 11, 13, 14





