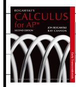


3.11 (Day 1) Related Rates

Objective:
1. Solve problems involving related rates.

DERIVATIVE → SLOPE → RATE OF CHANGE

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{\text{CHANGE IN } y}{\text{CHANGE IN } x}$$

$$\therefore \quad \frac{dw}{dt} = \frac{\text{CHANGE IN } w}{\text{CHANGE IN } t}$$

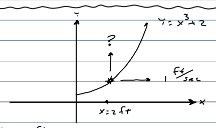
FIND THE DERIVATIVE WITH RESPECT TO x

$$y = 2x^3 + 1$$
$$\frac{dy}{dx} = 6x^2 \cancel{\frac{dx}{dx}} \quad 1$$

FIND THE DERIVATIVE WITH RESPECT TO t

$$y = 2x^3 + 1$$
$$\frac{dy}{dt} = 6x^2 \frac{dx}{dt}$$

1.



$\frac{dx}{dt} = 1$ ft/sec

$\frac{dy}{dt} = ?$

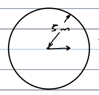
$x = 2$ ft

$$y = x^3 + 2$$
$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$
$$\frac{dy}{dt} = 3(2)^2(1)$$
$$\frac{dy}{dt} = 12 \text{ ft/sec}$$

RELATED RATES

1. FIND AN EQUATION THAT RELATES THE RATES
2. IDENTIFY ALL VARIABLES AND DRAW A PICTURE
3. DIFFERENTIATE
4. SUBSTITUTE
5. SOLVE.

2.



RADIUS IS INCREASING AT 2 m/s

HOW FAST IS THE AREA CHANGING

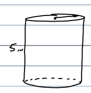
$\frac{dr}{dt} = 2$ m/s

$\frac{dA}{dt} = ?$

$r = 5$ m

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$
$$\frac{dA}{dt} = 2\pi(5)(2)$$
$$\frac{dA}{dt} = 20\pi \frac{m^2}{s}$$

3.



RADIUS IS INCREASING AT 2 in/sec

RADIUS = 4 in

HOW FAST IS THE VOLUME CHANGING?

$\frac{dr}{dt} = 2$ in/sec

$r = 4$ in

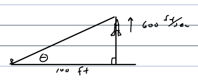
$\frac{dV}{dt} = ?$

$h = 5$ in

CONSTANT

$$V = \pi r^2 h$$
$$V = \pi r^2(5)$$
$$V = 5\pi r^2$$
$$\frac{dV}{dt} = 10\pi r \frac{dr}{dt}$$
$$\frac{dV}{dt} = 10\pi(4)(2)$$
$$\frac{dV}{dt} = 80\pi \frac{in^3}{sec}$$

4.



HOW FAST IS THE ANGLE OF ELEVATION CHANGING AT A HEIGHT OF 100 ft.?

$x = 100$ ft

$y = 100$ ft

$\frac{dy}{dt} = 600 \frac{ft}{sec}$

$\frac{d\theta}{dt} = ?$

$$\tan\theta = \frac{y}{x}$$
$$\tan\theta = \frac{y}{100}$$
$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{100}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta \frac{1}{100}\frac{dy}{dt}$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 \frac{1}{100} 600$$
$$= \frac{1}{2} \frac{600}{100}$$
$$\frac{d\theta}{dt} = 3 \frac{\text{RADIANS}}{\text{sec}}$$

Pg 199 # 1-9, 12, 13, 16, 17, 33

Aug 23-2:30 PM