CAPITAL HIGH SCHOOL

Calculus BC

2023/2024 Course Syllabus

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MATERIALS:	Calculus Book, Scientific Calculator

Course Overview:

Course Description: 3406 CALCULUS, LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS (Sr) (1 Credit)

This is a one-year, integrated mathematics course. Topics include limits and continuity, differentiation, integration, sequences, finite differences, finite-difference equations in dynamical systems, and differential equations. There is heavy emphasis on applications, mathematical modeling, and problem-solving techniques.

Textbook:

Main/Required Text

1) Rogawski, Jon. Single Variable Calculus: Early Transcendentals. New York, New York: W.H. Freeman and Company, 2012.

Supplementary Texts

In class, we will use several additional textbooks, and supplements to further students' conceptual development.

2) Larson, Hostetler, & Edwards. Calculus with Analytic Geometry, Seventh Edition. Lexington, MA: D.C. Heath and Company.
3) Hughes-Hallett, Gleason, McCallum, et al. Calculus Fifth Edition. Danvers, MA: John Wiley & Sons, Inc.

Technology:

Technology is an essential staple in the curriculum, used daily. Students are required to have a calculator from the TI-83 series, the TI-84 series, or above. In part, students use graphing calculators to confirm and/or interpret results of explorations and experiments and to support conclusions obtained using symbolic manipulation.

Course Goals:

- Extend knowledge and understanding of mathematical concepts, specifically in the study of differential and integral calculus.
- Develop skills in formulating and solving problems involving differential and integral calculus.
- Design mathematical models using calculus and basic differential equations to capture the essence of real-world patterns and phenomena.
- Interpret mathematical models and their solutions in the context of their real-world applications.
- Critique mathematical models to identify their strengths and weaknesses and modify them to make them better models.
- Expand knowledge and understanding of the world through mathematical analysis.
- Develop skills to effectively use modern computing, information, and communication technologies.
- Develop skills to effectively communicate mathematical ideas orally and in writing.

Class Rules: The Bruin Basics (standard rules of Capital High) are posted in the classroom.

Grading Scale:

91 – 100 = A	78 – 79.99 = C+	62 – 67.99 = D
90 – 91.99 = A-	72 – 77.99 = C	60 – 61.99 = D-
88 – 89.99 =B+	70 – 71.99 = C-	59.99 AND BELOW = F
82 – 87.99 = B	68 – 69.99 = D+	

Course Outline

Unit 1 – Limits (Chapter #2)

Asymptotic and unbounded behavior

- Understand asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Find and compare different rates of change. Understand the difference between secant lines and tangent lines

Limits of functions (including one-sided limits)

- Have an intuitive understanding of the limiting process
- Calculate limits using algebra
- Estimate limits from graphs or tables of data
- Calculate Trigonometric Limits
- With a basic review of differentiation apply L'Hopital's Rule to evaluate limits
- Understand when a limit does not exist or is infinite

Continuity as a property of functions

- Have an intuitive understanding of the different types of continuity
- Understand continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem)

Unit 2 – Derivatives (Chapters #3, 4)

Chapter #3 Differentiation

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear
- Instantaneous rate of change as the limit of average rate of
- Approximate rate of change from graphs and tables of values

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule
- Implicit differentiation (including finding the derivative of an inverse function)

Chapter #4 Applications of the Derivative

Derivative as a function

- Corresponding characteristics of graphs of ƒ and ƒ'
- Relationship between the increasing and decreasing behavior of ƒ and the sign of ƒ'
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
- The Mean Value Theorem and its geometric interpretation (Rolle's Theorem)

Second derivatives

- Corresponding characteristics of the graphs of ƒ, ƒ', and ƒ''
- Relationship between the concavity of ƒ and the sign of ƒ ''

  Points of inflection as places where concavity changes

  Analysis of graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Newton's Method
- L'Hospital's Rule

Unit 3 – Integration (Chapters #5, 6, 7)

Chapter #5 The Integral

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions

  Numerical approximations to definite integrals
- Use of Riemann sums (Using left, right, midpoint evaluation points, and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values)

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval
- Basic properties of definite integrals (examples include additivity and linearity)

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Chapter #7 Techniques of Integration

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only), trigonometric substitutions
- Improper integrals (as limits of definite integrals)

Chapter (#6, 8, 9, 11) Applications of the Integral

Applications of integrals

- Finding the area of a region
- Volume of a solid with known cross sections
- Surface Area
- Average value of a function
- Distance traveled by a particle along a line
- The length of a curve (including a curve given in parametric form)
- Accumulated change from a rate of change

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
- Numerical solution of differential equations using Euler's method
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)
- Solving logistic differential equations and using them in modeling
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- Solving logistic differential equations and using them in modeling

Application of Parametric and Polar Equations

- Graph relations in parametric mode
- Eliminate the parameter to identify the function form of a parametric
- Find the slope of a tangent line to a curve in parametric mode
- Find the concavity of a curve in parametric mode
- Find the arc length of a curve in parametric mode.
- Find the position of an object in motion in two dimensions from its velocity
- Graph curves in polar form
- Recognize graphs from certain polar equations
- Determine and interpret intervals of increasing or decreasing of a polar curve
- Find slopes of a lines tangent to polar curves
- Find the arc length of an equation in polar form
- Find the area of a region in polar form

Unit 4 – Polynomial Approximation and Series (Chapter #8, 10)

Concept of series

- A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology will be used to explore convergence and divergence.

Series of constants

- Motivating examples, including decimal expansion
- Geometric series with applications
- The harmonic series
- Alternating series with error bound
- Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of

p-series

- Comparing series to test for convergence or divergence

Taylor series

- Taylor polynomial approximation with graphical demonstration of Convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
- Maclaurin series and the general Taylor series centered at x = a. Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.
- Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
- Functions defined by power series
- Radius and interval of convergence of power series
- Lagrange error bound for Taylor polynomials

Unit 5 - Discrete Dynamical Systems (After AP test)

Modeling Change with Difference Equations

- Discrete Change as a Difference Equation
- Exact Models of “Discrete” Change
- “Discrete” Versus “Continuous” Change
- Approximate Change

Numerical Solutions

- Long-Term Behavior
- Homogeneous Dynamical Systems when r is a Constant
- Nonhomogeneous Dynamical Systems when r is a Constant
- Finding Equilibrium Values.

First Order Linear Discrete Dynamical Systems

- Analytical Solutions and the Method of Conjecture
- First Order Linear Homogeneous Systems

- Nonhomogeneous Difference Equations
- Nonhomogeneous Systems: Exponential Functions
- Nonhomogeneous Systems: Polynomial Functions
- First-Order Linear Homogeneous Equations
- First-Order Linear Nonhomogeneous Equations
- Applications of First-Order, Linear Equations

Dynamical Systems of Several Variables

- Classifying Systems.
- Applications Involving Systems
- Analytical Solutions and Equilibria
- Second-Order Linear Discrete Dynamical Systems
- First-Order Linear Homogeneous Equations
- First-Order Linear Nonhomogeneous Equations
- Applications of First-Order, Linear Equations
- Eigenvalues and Eigenvectors